INTENSIFICATION OF CONVECTIVE HEAT TRANSFER IN CHANNELS WITH A POROUS HIGH-THERMAL-CONDUCTIVITY FILLER. II. FORCED HEAT-TRANSFER REGIME

V. A. Maiorov, V. M. Polyaev,
and L. L. Vasil'ev

UDC 536.24:532.685

Results are presented from an analytical study of intensification of convective single-phase heat transfer in a channel with a porous high-thermal-conductivity filler in the case of intensive external heating.

The investigation [1] presented results of analytical and experimental studies of heat transfer in a channel with a permeable, high-thermal-conductivity filler under conditions of local thermal equilibrium between the porous material and the single-phase heat carrier in the case of moderate thermal loads. Here, we continue this study for conditions of forced flow under high thermal loads in the case where the finiteness of the value of the volume rate h_V of interpore heat transfer becomes significant and there is a gradual increase in the temperature difference $T - t$ between the porous material and coolant. We will use the notation and scheme of sectional enumeration of the formulas and figures adopted in [1].

Forced heat transfer is distinguished by substantial mass rates of the heat carrier G and, thus, high values of the parameter $\mathrm{Pe} = G\delta c/\lambda$. Thus, as was shown in [1], in this case we can ignore the effect of axial heat transfer by conduction: $\lambda\partial^2T/\partial z^2 = 0$. In this instance ($\vartheta \neq \Theta$, $\partial^2\Theta/\partial\xi^2 = 0$), the system of equations (I.13)-(I.14) takes the form:

$$\frac{\partial^2\Theta}{\partial\zeta^2} = \gamma^2(\Theta - \vartheta); \tag{II.1}$$

$$\mathrm{Pe}\,\frac{\partial\vartheta}{\partial\xi} = \gamma^2(\Theta - \vartheta). \tag{II.2}$$

Three boundary conditions are required for this case — of the five conditions written earlier, for example (I.4)-(I.8), (I.4) and (I.8) are omitted. Here, if we seek $\vartheta(\xi, \zeta)$ in the form $\vartheta(\xi, \zeta) = \varphi(\zeta)\psi(\xi)$, then Eq. (I.16) for determining $\varphi(\xi)$ remains unchanged, while Eq. (I.17) is simplified:

$$\mathrm{Pe}(1 + 4\mu^2/\gamma^2)\,\psi' + 4\mu^2\psi = 0.$$

<u>Boundary Conditions of the First and Third Kind.</u> The solution of system (II.1)-(II.2), with boundary conditions (I.5)-(I.7), has the form

$$\vartheta = 2\sum_{1}^{\infty}\frac{A_n\mu_n}{\sin\mu_n}\cos(2\mu_n\zeta)\exp[-4\mu_n^2\xi/\mathrm{Pe}(1 + 4\mu_n^2/\gamma^2)]; \tag{II.3}$$

$$\Theta = 2\sum_{1}^{\infty}\frac{A_n\mu_n\cos(2\mu_n\zeta)}{\sin\mu_n(1 + 4\mu_n^2/\gamma^2)}\exp[-4\mu_n^2\xi/\mathrm{Pe}(1 + 4\mu_n^2/\gamma^2)]; \tag{II.4}$$

$$\bar{\vartheta} = 2\sum_{1}^{\infty}A_n\exp[-4\mu_n^2\xi/\mathrm{Pe}(1 + 4\mu_n^2/\gamma^2)]; \tag{II.5}$$

$$\mathrm{Nu}_k^{\Delta} = \frac{4}{\bar{\vartheta}}\sum_{1}^{\infty}\frac{A_n\mu_n^2}{(1 + 4\mu_n^2/\gamma^2)}\exp[-4\mu_n^2\xi/\mathrm{Pe}(1 + 4\mu_n^2/\gamma^2)]. \tag{II.6}$$

Novolipetsk Polytechnic Institute. Moscow Higher Technical School. A. V. Lykov Institute of Heat and Mass Transfer, Academy of Sciences of the Belorussian SSR, Minsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 47, No. 2, pp. 199-205, August, 1984. Original article submitted May 31, 1983.

 0022-0841/84/4702-0886$08.50

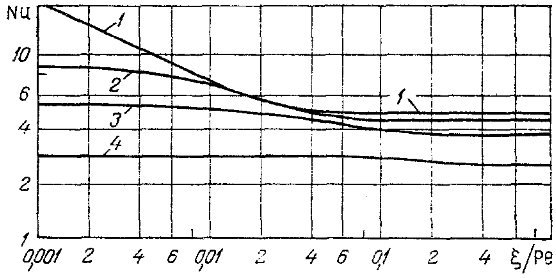

Fig. II.1. Effect of rate of interpore heat transfer (γ^2) on the change in the local heat-transfer criterion on the inlet section of a permeable matrix in a planar channel wall temperature (Bi $\to \infty$): 1) $\gamma^2 \to \infty$; 2) $\gamma^2 = 100$; 3) 31.6; 4) 10.

The coefficients A_n in Eqs. (II.3)-(II.6), as in (I.20), are the same as for the problem with local thermal equilibrium inside the porous material (T = t), while the characteristic values μ_n are found from the same characteristic equation (I.22). A constant wall temperature $T_w = t_\infty$ (Bi $\to \infty$) corresponds to the values $\mu_n = (2n - 1)\pi/2$, n = 1, 2, 3, ..., $A_n = 1/\mu_n^2$.

The solution of (II.3)-(II.6) differs from the results (I.19)-(I.26) with local thermal equilibrium (T = t) for the limiting case without allowance for axial heat conduction (Pe $\to \infty$, $B_n = 4\mu_n^2/\text{Pe}$) by the appearance of the coefficient $(1 + 4\mu_n^2/\gamma^2)$, which considers the effect of the finiteness of h_V (volume rate of interpore heat transfer), $\gamma^2 = h_V\delta^2/\lambda$. With $\gamma^2 \to \infty$, when $(1 + 4\mu_n^2/\gamma^2) \to 1$, we have t $\to$ T. Thus, analysis of the results (II.3)-(II.6) is best done to explain the effect of the parameter γ^2 with a reduction in it from $\gamma^2 = \infty$.

In Fig. II.1, the results deviate by 1% from the limiting case ($\gamma^2 = \infty$) at $\gamma^2 = 1000$. With a further decrease in γ^2, the rate of heat transfer from the channel wall decreases both on the inlet section and in the region of stabilized heat transfer.

Using (II.6) we can obtain the following for the region of stable heat transfer

$$\text{Nu}^\Delta_{k\infty}/\text{Nu}_{k\infty} = 1/(1 + 2\,\text{Nu}_{k\infty}/\gamma^2). \tag{II.7}$$

The analogous expression for a circular channel is

$$\text{Nu}^\Delta_{k\infty}/\text{Nu}_{k\infty} = 1/(1 + 4\text{Nu}_{k\infty}/\gamma^2). \tag{II.8}$$

These relations were used to construct the curves in Fig. II.2, which reflect the reduction in the heat-transfer rate $\text{Nu}^\Delta_{k\infty}$ in the stable heat-transfer region with a finite value of γ^2 compared to the value of $\text{Nu}_{k\infty}$ corresponding to local thermal equilibrium (T = t) inside the permeable matrix ($\gamma^2 = \infty$).

It is easy to use (II.7) to determine the limiting value of γ^2 at which we should begin to consider the effect of the finiteness of the volumetric heat-transfer rate h_V on the reduction in heat transfer from the channel wall to the heat carrier flowing inside the porous material. For example, on the condition that the ratio $\text{Nu}^\Delta_\infty/\text{Nu}_\infty$ decreases by no more than a small amount ε, we should have the following: $\gamma^2 > 2\text{Nu}_\infty/\varepsilon$ for the planar channel and $\gamma^2 > 4\text{Nu}_\infty/\varepsilon$ for the circular channel.

It also follows from the data in Fig. II.1 that a reduction in γ^2 is accompanied by an increase in the length ξ^Δ_l of the initial thermal section, the value of the latter for the planar channel being calculated from the expression:

$$\frac{\xi_l^\Delta}{\text{Pe}} = \frac{1}{4}\left[\frac{\mu_2^2}{1 + 4\mu_2^2/\gamma^2} - \frac{\mu_1^2}{1 + 4\mu_1^2/\gamma^2}\right]^{-1} \ln\left[100\,\frac{\mu_2^2 A_2 (1 + 4\mu_1^2/\gamma^2)}{\mu_1^2 A_1 (1 + 4\mu_2^2/\gamma^2)}\right]. \tag{II.9}$$

This property is clearly manifest from the data in Fig. II.3, where the quantity ξ^Δ_l is represented in proportion to the quantity ξ_l, corresponding to the limiting value $\gamma^2 = \infty$.

Boundary Conditions of the Second Kind. With boundary conditions (I.6), (I.11), and (I.12) the solution of system (II.1)-(II.2) has the form:

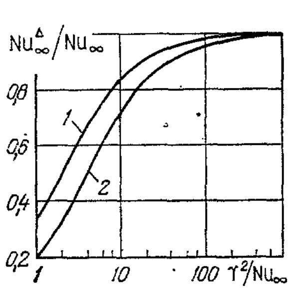

Fig. II.2

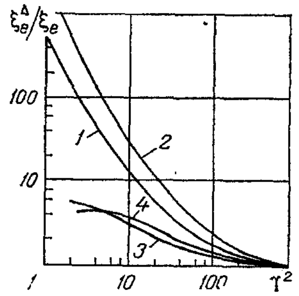

Fig. II.3

Fig. II.2. Reduction in heat-transfer rate in a channel with a porous filler during a change in the rate of interpore heat transfer: 1) planar channel; 2) circular channel.

Fig. II.3. Change in the length of the initial thermal section in a channel with a porous filler during a change in the rate of interpore heat transfer: 1) planar channel, T_w = const; 2) circular channel, T_w = const; 3) planar channel, q_w = const; 4) circular channel, q_w = const.

$$\vartheta = \frac{2}{\mathrm{Pe}}\xi + \zeta^2 - \frac{1}{12} - \sum_{1}^{\infty} \frac{(-1)^n}{\mu_n^2}\cos(2\mu_n\zeta)\exp[-4\mu_n^2\xi/\mathrm{Pe}(1+4\mu_n^2/\gamma^2)]; \tag{II.10}$$

$$\Theta = \frac{2}{\mathrm{Pe}}\xi + \zeta^2 - \frac{1}{12} + \frac{2}{\gamma^2} - \sum_{1}^{\infty} \frac{(-1)^n\cos(2\mu_n\zeta)}{\mu_n^2(1+4\mu_n^2/\gamma^2)}\exp[-4\mu_n^2\xi/\mathrm{Pe}(1+4\mu_n^2/\gamma^2)]; \tag{II.11}$$

$$\mathrm{Nu}^\Delta = (\Theta_w - \bar{\vartheta})^{-1} = \left\{\frac{1}{6} + \frac{2}{\gamma^2} - \sum_{1}^{\infty} \frac{\exp[-4\mu_n^2\xi/\mathrm{Pe}(1+4\mu_n^2/\gamma^2)]}{\mu_n^2(1+4\mu_n^2/\gamma^2)}\right\}^{-1}. \tag{II.12}$$

All of the observations made with regard to the effect of the parameter γ^2 on the heat-transfer characteristics in channels with a porous filler in the absence of thermal equilibrium and boundary conditions of the first and third kind are also valid for the case of boundary conditions of the second kind. This follows, for example, from comparison of the data in Fig. II.1 and Fig. II.4.

In the region of stable heat transfer, we obtain the following relations for planar and circular channels

$$\mathrm{Nu}^\Delta_\infty = \frac{6}{1+12/\gamma^2}; \quad \mathrm{Nu}^\Delta_\infty = \frac{8}{1+32/\gamma^2} \tag{II.13}$$

being special cases of (II.7) and (II.8) with Nu_∞ = 6 and Nu_∞ = 8, respectively.

The length ξ^Δ_l of the initial thermal section in the planar channel is determined from the expression

$$\frac{\xi_l^\Delta}{\mathrm{Pe}} = \frac{1+4\mu_1^2/\gamma^2}{4\mu_1^2}\ln\left[\frac{600}{\mu_1^2(1+4\mu_1^2/\gamma^2)}\right], \tag{II.14}$$

in which $\mu_1 = \pi$.

It follows from the data in Fig. II.3 that with a constant external heat flow a reduction in γ^2 causes a less substantial increase in the length of the initial section than in the case of boundary conditions of the first kind.

It is also interesting to note certain results for the temperature fields on the section of stabilized heat transfer ($\xi > \xi^\Delta_l$). It follows from (II.10)-(II.11) that the temperature of the coolant increases linearly at any point of the channel cross section $\vartheta = 2\xi/\mathrm{Pe} + \xi^2 - 1/12$, while the temperature difference $\Theta - \vartheta = 2/\gamma^2$ remains constant both along and across the channel. Meanwhile, its absolute value is easily found through the principal characteristics of the process: $T - t = 2q_w/h_v\delta$.

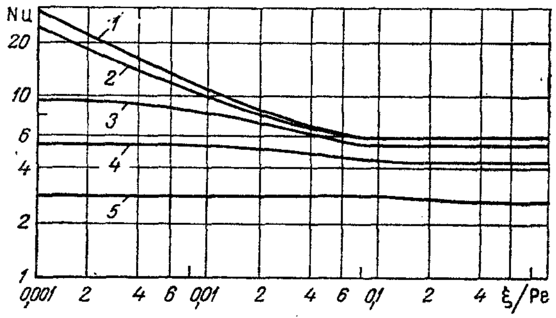

Fig. II.4. Effect of the rate of interpore heat transfer (γ^2) on the change in the local criterion of heat transfer on the inlet section of a permeable matrix in a planar channel with a constant external heat flow q_w = const; 1) $\gamma^2 = \infty$; 2) $\gamma^2 = 1000$; 3) 100; 4) 31.6; 5) 10.

Evaluation of the Effectiveness of Using a Porous Filler in Channels. Most well-known methods of intensifying heat transfer in channels lead to an increase in hydraulic resistance. Here, depending on the criterion used to evaluate the effectiveness of the intensification, a positive result is obtained for a given heat exchanger when a certain relation is maintained between the ratios of the Nusselt numbers $\overline{Nu}^*/\overline{Nu}_0$ and the drag coefficients ξ^*/ξ_0 for channels with intensification ($\overline{Nu}^*$, ξ^*) and without it ($\overline{Nu}_0$, ξ_0). Thus for example, it was shown in [2] that in the intensification of heat transfer in a turbulent flow in the channels of tubular heat exchangers, a positive effect, evaluated by three different criteria, is obtained with the satisfaction of the exponential relation $\xi^*/\xi_0 < (\overline{Nu}^*/\overline{Nu}_0)^{3.5}$.

Let us examine the change in the mean heat-transfer coefficient $\bar{\alpha}^*/\bar{\alpha}_0$ and drag coefficient ξ^*/ξ_0 on the inlet section of a planar channel of width δ during the motion of a single-phase heat carrier with a thermal conductivity λ_0 and a number Pr_0 as a result of the filling of the channel by a porous material with a thermal conductivity λ. The material has a viscosity coefficient α, an inertial drag coefficient β, and a mean particle size d_p. The mass rate of the heat carrier G and the Reynolds number of the flow $Re = G\delta/\mu$ remain constant.

An increase in the heat-transfer rate in the channel when it is filled with a permeable matrix is proportional to the ratio of the thermal conductivities λ/λ_0 of the porous material and heat carrier:

$$\frac{\bar{\alpha}^*}{\bar{\alpha}_0} = \frac{\lambda}{\lambda_0}\,\frac{\overline{Nu}^*}{\overline{Nu}_0}\,. \tag{II.15}$$

Here, $\overline{Nu}^*$ is the criterion of mean heat transfer in the channel with the filler, while the criterional equation for heat transfer in a channel without a filler $\overline{Nu}_0 = \overline{Nu}_0\,(Re, Pr_0, l/\delta)$ is chosen in relation to the flow regime. It follows from the example shown in Fig. II.5 that the use of a porous filler is most effective in the laminar flow regime, when the quantity $\overline{Nu}^*/\overline{Nu}_0$ may become greater than unity. It decreases with an increase in the Reynolds number. However, the ratio λ/λ_0 is fairly easily monitored and may reach an appreciable value, especially in the flow of gaseous heat carriers. For example, for air λ_0 = 0.032 W/(m•K) and for porous metal with a limiting value λ = 32 W/(m•K) we have λ/λ_0 = 1000. Thus, an increase in the heat-transfer rate under these conditions $\bar{\alpha}^*/\bar{\alpha}_0 \simeq 1000$ is possible at least with low flow rates for the heat carrier. A particularly large value of λ/λ_0 may be reached at cryogenic temperatures 5-40°K, when the thermal conductivity of high-purity copper and aluminum increases by almost one order and reaches $\lambda \simeq 4000$ W/(m•K).

The drag coefficient of the inlet section of a planar channel in the case of laminar flow is calculated from the formula [4]: $\xi_0 = (24/Re + 0.61\ \delta/l)$. The equation in [5] for calcualting the resistance of a porous material when the characteristic dimension δ is used in the Reynolds number takes the form $\xi^* = [2/Re + 2(\beta/\alpha)/\delta]\delta^2\alpha$. Then the sought value of the ratio of the drag coefficients is

$$\frac{\xi^*}{\xi_0} = \frac{[2/Re + 2\,(\beta/\alpha)/\delta]}{(24/Re + 0.61\,\delta/l)}\,\delta^2\alpha = K\,(Re)\,\delta^2\alpha. \tag{II.16}$$

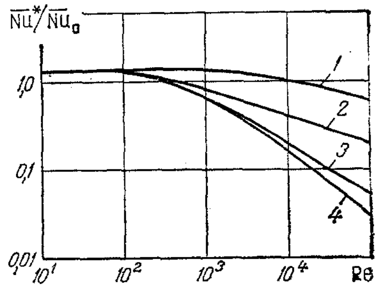

Fig. II.5. Dependence of the ratio of the heat-transfer criteria in a planar channel with a porous filler ($\overline{Nu}^*$) and without the filler ($\overline{Nu}_0$) on the Reynolds number Re of the flow ($l/\delta = 10$; $Pr_0 = 1$; $q_w = const$): 1) $\lambda/\lambda_0 = 1$; 2) 10; 3) 100; 4) 1000.

Here, K(Re) is a slightly varying function. It follows from this that when the channel is filled with a permeable matrix, the drag coefficient increases in proportion to the value of $\delta^2\alpha$, or, considering that $\alpha \sim d_p^{-2}$, in proportion to $(\delta/d_p)^2$, the square of the ratio of the channel width to the mean size of the particles of the porous material. Evaluation of this ratio for actual values $\delta = 3.5$ mm and $\alpha = 10^{10}$ m^{-2} gives a value $\xi^*/\xi_0 \simeq 10^4$.

Thus, the above method of intensifying heat transfer in channels differs from other well-known methods mainly in the substantial increase in both heat transfer $\overline{\alpha}^*/\overline{\alpha}_0$ and drag ξ^*/ξ_0. The positive effect of intensification will be achieved for each specific case when there is a certain relationship between these two quantities. For example, it was established experimentally in [6] that filling a channel with a porous netted metal does not increase the amount of power expended on pumping the heat carrier compared to a smooth channel for the given prescribed thermal state of the heated channel wall.

The widest application of porous metal inserts for intensifying heat transfer in channels will be found under special conditions such as: when it is necessary to make the heat exchanger as small as possible; for sections with an extremely high heat flux; in the case of low rates of flow and limited reserves of the heat carrier; with a large available pressure gradient, as well as to force the complete evaporation and condensation of coolant flows.

Aspects of heat transfer and resistance in channels with a porous high-thermal-conductivity filler during evaporation of the flow of heat carrier were examined in [7, 8].

LITERATURE CITED

1. V. A. Maiorov, V. M. Polyaev, L. L. Vasil'ev, and A. I. Kiselev, "Intensification of convective heat exchange in channels with a porous high-thermal-conductivity filler. I. Heat transfer with local thermal equilibrium inside the porous material," Inzh.-Fiz. Zh., 47, No. 1.
2. E. K. Kalinin, G. A. Dreitser, and S. A. Yarkho, Intensification of Heat Exchange in Channels [in Russian], Mashinostroenie, Moscow (1981).
3. L. A. Novitskii and I. G. Kozhevnikov, Thermophysical Properties of Materials at Low Temperatures (Handbook) [in Russian], Mashinostroenie, Moscow (1975).
4. B. S. Petukhov, Heat Exchange and Resistance in the Laminar Flow of Liquids in Pipes [in Russian], Énergiya, Moscow (1967).
5. S. V. Belov, Porous Metals in Machine Construction [in Russian], Mashinostroenie, Moscow (1981).
6. V. M. Polyaev, L. L. Morozova, E. V. Kharybin, and N. I. Avraamov, "Intensification of heat transfer in an annular channel," Izv. Vyssh. Uchebn. Zaved., Mashinostr., No. 2, 86-90 (1976).
7. V. A. Maiorov, "Structure and resistance of a two-phase evaporating flow in porous materials," Izv. Akad. Nauk SSSR, Energ. Transp., No. 5, 126-133 (1980).
8. V. A. Maiorov, "Intensification of heat transfer in the evaporation of a flow in a channel with a porous high-thermal-conductivity filler," Prom. Teplotekhnika, 3, No. 4, 22-28 (1981).